Exam

Name_____

1) Under what cindition on a and b, tge given equation is exact: 1) _____

$(axy^2 + by)\, dx + (bx^2y + ax)dy = 0.$

A) $a = 1, b = 2$

B) $a = - b$

C) $a = b$

D) $a = 2b$

E) $5a = 3b$

2) The order of the equation $\frac{d^4y}{dx^4} + \frac{d^3y}{dx^3} + \left(\frac{d^2y}{dx^2}\right)^4 + e^x y^{10} = x + 19$ is 2) _____

A) 10 B) 19 C) 4 D) 3 E) 8

3) The solution of the following IVP is : 3) _____

$$\begin{cases} yy' + 36x = 0 \\ y(0) = 1 \end{cases}$$

A) $y^2 + 36x^2 = 1$

B) $y^2 + x^2 = 1$

C) $36y^2 + x^2 = 1$

D) $y^2 + 36x^2 = 36$

E) $y^2 + x^2 = 36$

4) We are given the equation : $\frac{dy}{dx} - \frac{2}{x} y = \frac{1}{x^2} y^2$. Which one is true : 4) _____

A) It is a first - order linear ode

B) The substitution $u = y^{-2}$ is helpful to solve it

C) The substitution $u = y^{-1}$ is helpful to solve it

D) It is an exact equation

E) $y = x$ is a solution

5) We are given $4y'' + 12y' + 9y = 0$. Then the characteristic equation is 5) _____

A) $4\lambda^2 + 12\lambda + 9\ \lambda = 0$

B) $4\lambda^2 + 12\lambda + 9 = 0$

C) $4\lambda^2 + 12\lambda = 0$

D) $4\lambda^2 + 12\lambda = 9$

E) $\lambda^2 + 12\lambda + 9 = 0$

6) Let c_1 and c_2 be constants. Then which one of the following is the general solution to $y'' - y' - 2y = 0$ 6) _____

A) $y = \left(c_1 + c_2 x\right)\ e^{-x}$

B) $y = c_1\ e^{2x} + c_2\ e^{2x}$

C) $y = c_1\ e^{-x} + c_2\ e^{-x}$

D) $y = e^x \left(c_1\ \cos x + c_2 \sin x\right)$

E) $y = c_1\ e^{2x} + c_2\ e^{-x}$

7) Let c_1 and c_2 be constants. Then which one of the following is the general solution to $y'' + 6y' + 9y = 0$ 7) _____

A) $y = \left(c_1 + c_2 x\right)\ e^{3x}$

B) $y = \left(c_1 + c_2 x\right)\ e^{x}$

C) $y = e^x \left(c_1\ \cos 3x + c_2 \sin 3x\right)$

D) $y = c_1\ e^{3x} + c_2\ e^{-3x}$

E) $y = \left(c_1 + c_2 x\right)\ e^{-3x}$

8) Let c_1 and c_2 be constants. Then which one of the following is the general solution to $y'' + 16y = 0$ 8) _____

A) $y = \left(c_1 + c_2 x\right) e^{4x}$

B) $y = e^{4x}\left(c_1 \cos x + c_2 \sin x\right)$

C) $y = c_1 \cos 4x + c_2 \sin 4x$

D) $y = e^{x}\left(c_1 \cos 4x + c_2 \sin 4x\right)$

E) $y = c_1 e^{4x} + c_2 e^{-4x}$

9) **Bonus :** Let C be a constant. Then the general solution of the following equation is: $y' + 2y = e^{x}$ 9) _____

A) $y(x) = \frac{1}{3} e^{x} + C e^{-2x}$

B) $y(x) = \frac{1}{3} e^{5x} + C e^{-2x}$

C) $y(x) = \frac{1}{3} e^{5x} + C e^{2x}$

D) $y(x) = \frac{1}{3} e^{-2x} + C e^{x}$

E) $y(x) = e^{x} + C e^{-2x}$

Solve the following **first order linear equation (2pts)** :

$$x\,y' = x^2 + 3y\,, \qquad x > 0$$

Hint: First rewrite the equation in the standart form: $\boldsymbol{y' + p(x)\,y = q(x)}$

Verify that the following equation is **exact (1 pt)** and then solve it **(2 pts).**

$$\left(\frac{1}{x} + 2y^2x\right)dx + \left(2yx^2 - cos(y)\right)dy = 0$$

We are given the equation : $x^2\, y" - x\, y' + y = 0$. By knowing that $y = x$ is a solution, find an independent solution by method of reduction **(2.5 pts).** then find the general solution of the equation. **(0.5 pt).**